Systems and Pulleys

- A **system** is when two or more masses are all connected to each other
  - Applying a force to one object will cause all objects to accelerate in the same direction with the same magnitude

- For any system and any other force problem, it is necessary to draw a free-body diagram
- When creating a force net equation, imagine the problem to be horizontal by swinging out the masses and the forces attached to those masses.
  - This is just a strategy to solve the problems, it is not the actual free-body diagram

**Atwood’s Machine**

\[
\begin{align*}
T & \quad\quad F_{net} \\
F_{g1} & \quad m_1 \\
F_{net} & \quad T \\
F_{g2} & \quad m_2
\end{align*}
\]

\[
F_{net} = F_{g2} + ( - F_{g1})
\]

**Fletcher’s Trolley**

\[
\begin{align*}
F_{f} & \quad\quad F_{net} \\
F_{g1} & \quad m_1 \\
F_{net} & \quad F_{f} \\
F_{g2} & \quad m_2
\end{align*}
\]

\[
F_{net} = F_{g2} + ( - F_{f})
\]
EXAMPLES:

1. Two spheres of masses 2.6 kg and 4.1 kg are tied together by a light string looped over a frictionless pulley.
   a. What will the acceleration of each mass be?

   \[ \Rightarrow F_{\text{net}} \leftarrow \{ m_2 \} \quad \text{O} \quad \Rightarrow F_{\text{net}} \]

   \[ \therefore F_{\text{net}} = F_{g_2} + (-F_{g_1}) \]

   \[ \begin{align*}
   M_{\text{total}} & = m_2 g - m_1 g \\
   a & = \frac{m_1 g - m_2 g}{m_{\text{total}}} = \frac{(2.6 \text{ kg})(9.8 \text{ m/s}^2) - (2.6 \text{ kg})(9.8 \text{ m/s}^2)}{2.6 \text{ kg} + 4.1 \text{ kg}} \\
   a & = 2.196 \text{ m/s}^2
   \end{align*} \]

   \[ m_1: a = 2.2 \text{ m/s}^2, \text{ up} \]
   \[ m_2: a = 2.2 \text{ m/s}^2, \text{ down} \]

   b. What is the tension in the string?

   * only look at one mass! *

   \[ m_1 \]

   \[ \begin{align*}
   \text{Fnet} & = T + (-F_g) \\
   \text{Fnet} + F_g & = T \\
   m_1 a + m_1 g & = T \\
   (2.6 \text{ kg})(2.196 \text{ m/s}^2) + (2.6 \text{ kg})(9.8 \text{ m/s}^2) & = T \\
   31.216 & = T
   \end{align*} \]

   \[ 31.216 = T \]
2. A 2.0 kg mass placed on a level table and is attached by a string passing over the edge of a table as illustrated in the diagram. The coefficient of friction between the 2.0kg mass and the table surface is 0.12.

a. Calculate the magnitude of acceleration of the system.

\[ F_{\text{net}} = m_1a + m_2g - \mu m_1 F_N \]

\[ a = \frac{m_2g - \mu m_1 F_N}{m_1} \]

\[ a = \frac{(5.0 \text{ kg})(9.8 \text{ m/s}^2) - (0.12)(19.62 \text{ N})}{(5.0 \text{ kg} + 2.0 \text{ kg})} \]

\[ a = 6.67 \text{ m/s}^2 \]

\[ \boxed{a = 6.7 \text{ m/s}^2} \]

b. Calculate the tension in the string.

* only look at one mass! *

\[ F_{\text{net}} = F_{\text{g2}} + (-T) \]

\[ T = m_2g - m_2a \]

\[ T = (5.0 \text{ kg})(9.8 \text{ m/s}^2) - (5.0 \text{ kg})(6.67 \text{ m/s}^2) \]

\[ T = 15.696 \text{ N} \]

\[ \boxed{T = 16 \text{ N}} \]
3. A 3000 kg truck is on a cliff and is tied with a rope to a 78.0 kg person hanging over the cliff. If the cliff is very icy (frictionless), what is the magnitude of acceleration of the system?

\[
\begin{align*}
\text{Net force:} & \quad F_{\text{net}} = F_g \\
\text{Free body diagram:} & \quad m_1, m_2, F_n, F_g \\
\text{Equations:} & \quad m_1a = m_2g \\
& \quad a = \frac{m_2g}{m_1} \\
& \quad a = \frac{78.0 \text{ kg}}{3000 \text{ kg}} \times 9.8 \text{ m/s}^2 (\text{gravitational force}) \\
& \quad a = 0.2485 \ldots \text{ m/s}^2 \\
& \quad a = 0.249 \text{ m/s}^2 \\
\end{align*}
\]

***Now try pg. 94 #15 - 19 (#19 challenger) & worksheet***
Pulley Systems: Worksheet

1.) Determine the magnitude of acceleration of each mass.
   \[2.26 \text{m/s}^2\]

2.) The diagram shows a 1.8kg mass and a 1.2kg mass suspended by a vertical pulley.
   a. Determine the magnitude of the acceleration of each mass. \[1.96 \text{m/s}^2\]
   b. Determine the tension in the rope. \[14.1 \text{N}\]

3.) A 4.0kg oak block on a horizontal, rough oak surface is attached by a light string that passes over a light, frictionless pulley to a hanging 2.0kg object. The magnitude of the force of friction on the 4.0kg block is 11.8N.

   a. Calculate the magnitude of acceleration of the system. \[1.3 \text{m/s}^2\]
   b. Calculate the tension in the string. \[17 \text{N}\]

4.) A 6.70kg block on a horizontal, rough surface is attached by a light string that passes over a light, frictionless pulley to a hanging mass of 10.5kg. If the system is accelerating at 3.15m/s², what is the coefficient of friction between the block and the horizontal surface? \[0.743\]
Vertical Dynamics

- We can use dynamics and Newton's second law to solve vertical questions in addition to horizontal problems which we have seen up to now.
- Recall that weight is the same or equal to $F_g$ and $F_g=mg$.
- When an object accelerates in the vertical direction, it experiences a change in weight.
  - This is called "apparent weight".
  - Apparent weight can also be called tension, applied force, or force apparent or $F_n$ on a surface.
  - This change in weight is due to the fact that the acceleration of the object is no longer 9.81 m/s$^2$, but either increased or decreased.
- Remember that dynamic/force problems usually always need a free-body diagram and a $F_{net}$ equation.

**EXAMPLE**: Draw a free body diagram and create a force net equation for
- an elevator that is stopped. This will be the same for an elevator that is moving up or down with uniform motion.

\[
F_{apparent wt} - F_n = F_{net}
\]

\[
F_{net} = F_n + (-F_g)
\]

\[
\text{no acceleration}
\]

\[
F_g = F_n
\]

\[
\therefore \text{real weight} = \text{apparent weight}
\]

- an elevator that is accelerating up.

\[
F_{apparent wt} + F_{fret} = F_n + (-F_g)
\]

\[
F_{fret} = F_n + (-F_g)
\]
EXAMPLES:
1. A 75.0 kg person is in an elevator. What is the apparent weight of the student if the elevator is
   a. moving up at a constant velocity.
      \[ \text{Net force} = F_{\text{up}} + (-F_g) \]
      \[ 0 \text{ b/c no acceleration} \]
      \[ F_g = F_{\text{up}} \]
      \[ mg = F_{\text{up}} \]
      \[ (75.0 \text{ kg})(9.81 \text{ m/s}^2) = F_{\text{up}} \]
      \[ 736 \text{ N} = F_{\text{up}} \]

   b. accelerating down at 0.750 m/s^2.
      \[ \text{Let down be positive!} \]
      \[ \text{Net force} = F_{\text{down}} + (-F_{\text{up}}) \]
      \[ ma = mg - F_{\text{up}} \]
      \[ F_{\text{up}} = mg - ma \]
      \[ F_{\text{up}} = (75.0 \text{ kg})(9.81 \text{ m/s}^2) - (75.0 \text{ kg})(0.750 \text{ m/s}^2) \]
      \[ F_{\text{up}} = 680 \text{ N} \]

   * acceleration down \( \Rightarrow \) apparent weight less!
2. A person who is rock climbing and is tethered to a rope, has a weight of 600 N. At one point of the person's journey, she has an apparent weight of 585 N. What is the acceleration of the rock climber at this point on her journey? 

\[ \text{F}_{\text{net}} = \text{F}_{\text{g}} - T \]
\[ \text{ma} = \text{F}_{\text{g}} - T \]
\[ a = \frac{\text{F}_{\text{g}} - T}{m} \]

\[ a = \frac{600 \text{N} - 585 \text{N}}{61.16 \text{... kg}} \]
\[ a = 0.245 \text{ m/s}^2 \]
\[ \text{down} \]

\[ \text{F}_{\text{g}} = 600 \text{N} \]
\[ T = 585 \text{N} \]
\[ a = ? \]

3. A crane is used to lift a car up into the air. The car has an apparent weight of 12000 N while it is accelerating up at 0.850 m/s\(^2\). What is the mass of the car?

\[ \text{F}_{\text{net}} = T + (-\text{F}_{\text{g}}) \]
\[ \text{ma} = T - mg \]
\[ ma + mg = T \]
\[ m(a + g) = T \]

\[ m = \frac{T}{(a + g)} \]
\[ m = \frac{12000 \text{N}}{0.850 \text{m/s}^2 + 9.81 \text{m/s}^2} \]
\[ m = 1125.7 \text{... kg} \]

\[ m = 1.13 \times 10^3 \text{ kg} \]

***Now try pg. 88 #3a,b,e,f, 4a-d, 5-9, 13***